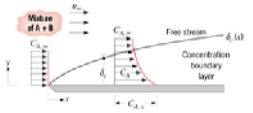
Introduction to Convection: Mass Transfer

The Concentration Boundary Layer

- Features
  - A consequence of evaporation or sublimation of species A from a liquid or solid surface across which a second fluid species B is flowing.
  - A region of the flow characterized by species fluxes and concentration gradients.
  - A region between the surface and free stream whose thickness δ_c increases in the flow direction.
  - Why does δ_c increase in the flow direction?
  - Manifested by a surface species flux $N''_{A,s}$ and a convection mass transfer coefficient h_m.
  - What is the heat transfer analog to Fick's law?



$$\delta_c \rightarrow \frac{C_{A,s} - C_A(y)}{C_{A,s} - C_{A,\infty}} = 0.99$$

$$\underbrace{N''_A = -D_{AB}\frac{\partial C_A}{\partial y}}_{\text{Fick's Law}}$$

$$N''_{A,s} = -D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0}$$

$$N''_{A,s} = h_m\left(C_{A,s} - C_{A,\infty}\right)$$

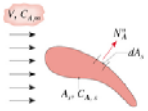
$$h_m = \frac{-D_{AB}\,\partial C_A/\partial y|_{y=0}}{C_{A,s} - C_{A,\infty}}$$

- Definitions

Term	Variable	Units
Species molar flux	$N''_{A,s}$	kmol/s·m^2
Species molar rate	$N_{A,s}$	kmol/s
Species mass flux	$n''_{A,s}$	kg/s·m^2
Species mass rate	n_A	kg/s
Species molar concentration	C_A	kmol/m^3
Species mass concentration (density)	ρ_A	kg/m^3
Species molecular weight	$\mathcal{M}_A$	kg/kmol
Convection mass transfer coefficient	h_m	m/s
Binary diffusion coefficient[1]	D_{AB}	m^2/s

[1] Table A.8

- Convection Calculations



Species Molar Flux:

$$N''_{A,s} = h_m\left(C_{A,s} - C_{A,\infty}\right)$$

Species Mass Flux:

$$n''_A = \mathcal{M}_A N''_A, \quad \rho_A = \mathcal{M}_A C_A$$

$$n''_{A,s} = h_m\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

Total Transfer Rates:

$$N_{A,s} = \bar{h}_m A_s\left(C_{A,s} - C_{A,\infty}\right)$$

$$n_{A,s} = \bar{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

Average Mass Transfer Coefficient:

$$\bar{h}_m = \frac{1}{A_s}\int_{A_s} h_m dx$$

Species Vapor Concentration or Density

- At a Vapor/Liquid or Vapor/Solid Interface

The vapor concentration/density corresponds to saturated conditions at the interface temperature T_s.

$$C_{A,s} = C_{A,sat}(T_s)$$

$$\rho_{A,s} = \rho_{A,sat}(T_s) = \mathcal{M}_A C_{A,sat}(T_s)$$

Assuming perfect gas behavior, the concentration/density may be estimated from knowledge of the saturation pressure.

$$C_{A,s} \approx \frac{p_{A,sat}(T_s)}{\mathcal{R}T_s} \approx \frac{\rho_{A,s}}{\mathcal{M}_A}$$

The concentration may also be directly determined from saturation tables. E.g., from Table A.6 for saturated water,

$$\rho_{A,s} = v_g^{-1}(T_s) = \mathcal{M}_A C_{A,s}$$

- Free Stream Conditions
  - The free stream concentration/density may be determined from knowledge of the vapor pressure, $p_{A,\infty}$, assuming perfect gas behavior.

$$C_{A,\infty} \approx \frac{p_{A,\infty}}{\mathcal{R}T_\infty} \approx \frac{\rho_{A,\infty}}{\mathcal{M}_A}$$

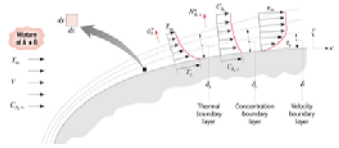
  - For water vapor in air, the free stream concentration/density may be determined from knowledge of the relative humidity, ϕ_∞.

$$\phi_\infty = \frac{p_{A,\infty}}{p_{A,sat}(T_\infty)} \approx \frac{C_{A,\infty}}{C_{A,sat}(T_\infty)} = \frac{\rho_{A,\infty}}{\rho_{A,sat}(T_\infty)}$$

For *dry air*,

$$\phi_\infty = 0$$

Species Boundary Layer Equation and Similarity



- Species boundary layer approximation:

$$\frac{\partial C_A}{\partial y} \gg \frac{\partial C_A}{\partial x}$$

- Species equation for a non-reacting boundary layer:

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

What is the physical significance of each term?

Is this equation analogous to another boundary layer equation?

- Dimensionless form of the species boundary layer equation:

$$C_A^* \equiv \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}}$$

$$u^*\frac{\partial C_A^*}{\partial x^*} + v^*\frac{\partial C_A^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L\, Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$$

$$Sc \equiv \frac{\nu}{D_{AB}} \rightarrow \text{the Schmidt number}$$

How may the Schmidt number be interpreted?

- Functional dependence for a prescribed geometry:

$$C_A^* = f\left(x^*, y^*, \mathrm{Re}_L, Sc\right)$$

$$h_m = \frac{-D_{AB}\,\partial C_A/\partial y\big|_{y=0}}{C_{A,s} - C_{A,\infty}} = -\frac{D_{AB}}{L}\frac{\left(C_{A,\infty} - C_{A,s}\right)}{\left(C_{A,s} - C_{A,\infty}\right)}\frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0} = \frac{D_{AB}}{L}\frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless local convection mass transfer coefficient is then

$$Sh \equiv \frac{h_m L}{D_{AB}} = \frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, \mathrm{Re}_L\, Sc\right)$$

$Sh \rightarrow$ local Sherwood number

What is the functional dependence of the average Sherwood number?

Analogies

- Heat and Mass Transfer Analogy

From analogous forms of the dimensionless boundary layer energy and species equations, it follows that, for a prescribed geometry and equivalent boundary conditions, the functional dependencies of Nu and Sh are equivalent.

$$Nu = f\left(x^*, \mathrm{Re}, \mathrm{Pr}\right)$$

$$Sh = f\left(x^*, \mathrm{Re}, Sc\right)$$

Since the Pr and Sc dependence of Nu and Sh, respectively, is typically of the form Pr^n and Sc^n, where n is a positive exponent ($0.30 \le n \le 0.40$),

$$\frac{Nu}{\mathrm{Pr}^n} = f\left(x^*, \mathrm{Re}\right) = \frac{Sh}{Sc^n}$$

$$\frac{hL/k}{\mathrm{Pr}^n} = \frac{h_m L/D_{AB}}{Sc^n}$$

$$\frac{h}{h_m} = \frac{k}{D_{AB}}\frac{\mathrm{Pr}^n}{Sc^n} = \frac{k}{D_{AB}}Le^{-n} = \rho c_p Le^{1-n}$$

$$Le = \frac{\alpha}{D_{AB}} \rightarrow \text{the Lewis number}$$

Analogies (cont.)

- Reynolds Analogy

For $dp^*/dx^* = 0$

$$C_f\frac{\mathrm{Re}}{2} = Nu$$

and $\mathrm{Pr} = Sc = 1$,

$$Nu = Sh$$

it follows that

$$\frac{C_f}{2} = \frac{Sh}{\mathrm{Re}} = St_m$$

where

$$St_m \equiv \frac{Sh}{\mathrm{Re}\,Sc} = \frac{h_m}{V} \rightarrow \text{the mass transfer Stanton number}$$

- Modified Reynolds Analogy

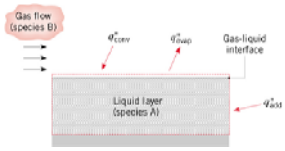
$$\frac{C_f}{2} = St_m Sc^{2/3} \equiv j_m \qquad 0.6 < Sc < 300$$

(j_m: Colburn j factor for mass transfer)

– Applicable to laminar flow if $dp^*/dx^* \sim 0$.

– Generally applicable to turbulent flow without restriction on dp^*/dx^*.

Evaporative Cooling



- The term evaporative cooling originates from association of the latent energy created by evaporation at a liquid interface with a reduction in the thermal energy of the liquid. If evaporation occurs in the absence of other energy transfer processes, the thermal energy, and hence the temperature of the liquid, must decrease.
- If the liquid is to be maintained at a fixed temperature, energy loss due to evaporation must be replenished by other means. Assuming convection heat transfer at the interface to provide the only means of energy inflow to the liquid, an energy balance yields

$$q''_{conv} = q''_{evap}$$

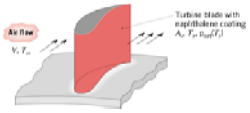
$$h\left(T_\infty - T_s\right) = n''_A h_{fg} = h_m\left[\rho_{A,sat}\left(T_s\right) - \rho_{A,\infty}\right]h_{fg}$$

$$\underbrace{\left(T_\infty - T_s\right)}_{\text{Steady-state Cooling}} = \underbrace{\left(h_m/h\right)}_{\text{Obtained from heat/mass transfer analogy}}\left[\rho_{A,sat}\left(T_s\right) - \rho_{A,\infty}\right]h_{fg}$$

- With radiation from the interface and heat addition by other means,

$$q''_{conv} + q''_{add} = q''_{evap} + q''_{rad}$$

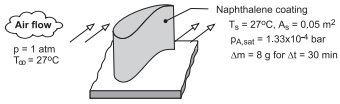
Problem 6.60: Use of the naphthalene sublimation technique to obtain the average convection heat transfer coefficient for a gas turbine blade.



KNOWN: Surface area and temperature of a coated turbine blade. Temperature and pressure of air flow over the blade. Molecular weight $\left(M_A = 128.16\ \text{kg/kmol}\right)$ and saturation vapor pressure of the naphthalene coating. Duration of air flow and corresponding mass loss of naphthalene due to sublimation.

FIND: Average convection heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy, (2) Negligible change in A_s due to mass loss, (3) Naphthalene vapor behaves as an ideal gas, (4) Solid/vapor equilibrium at surface of coating, (5) Negligible vapor density in freestream of air flow.

PROPERTIES: *Table A-4*, Air (T = 300K): $\rho = 1.161\ \text{kg/m}^3$, $c_p = 1007\ \text{J/kg·K}$, $\alpha = 22.5 \times 10^{-6}\ \text{m}^2/\text{s}$. *Table A-8*, Naphthalene vapor/air (T = 300K): $D_{AB} = 0.62 \times 10^{-5}\ \text{m}^2/\text{s}$.

ANALYSIS: From the rate equation for convection mass transfer, the average convection mass transfer coefficient may be expressed as

$$\bar{h}_m = \frac{n_A}{A_s\left(\rho_{A,s} - \rho_{A,\infty}\right)} = \frac{\Delta m/\Delta t}{A_s\ \rho_{A,s}}$$

where

$$\rho_{A,s} = \rho_{A,sat}\left(T_s\right) \approx \frac{\mathcal{M}_A\, p_{A,sat}}{\Re\, T_s} = \frac{\left(128.16\,\text{kg/kmol}\right)1.33\times10^{-4}\,\text{bar}}{0.08314\,\text{m}^3\cdot\text{bar/kmol}\cdot\text{K}\ \left(300\text{K}\right)} = 6.83\times10^{-4}\,\text{kg/m}^3$$

Hence,

$$\bar{h}_m = \frac{0.008\,\text{kg}/\left(30\,\text{min}\times60\,\text{s/min}\right)}{0.05\text{m}^2\left(6.83\times10^{-4}\,\text{kg/m}^3\right)} = 0.13\,\text{m/s}$$

Using the heat and mass transfer analogy with n = 1/3,

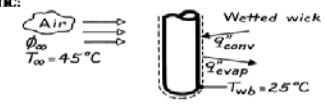
$$\bar{h} = \bar{h}_m\rho c_p Le^{2/3} = \bar{h}_m\rho c_p\left(\frac{\alpha}{D_{AB}}\right)^{2/3} = 0.130\,\text{m/s}\left(1.161\,\text{kg/m}^3\right)\times$$

$$1007\,\text{J/kg·K}\left(22.5\times10^{-6}/0.62\times10^{-5}\right)^{2/3} = 359\,\text{W/m}^2\cdot\text{K}$$

COMMENTS: The naphthalene sublimation technique is used extensively to determine convection coefficients associated with complex flows and geometries.

Problem 6.74: Use of wet and dry bulb temperature measurements to determine the relative humidity of an air stream.

SCHEMATIC:



ASSUMPTIONS: (1) Perfect gas behavior for vapor, (2) Steady-state conditions, (3) Negligible radiation, (4) Negligible conduction along thermometer.

PROPERTIES: *Table A-4*, Air (308K, 1 atm): $\rho = 1.135\ \text{kg/m}^3$, $c_p = 1007\ \text{J/kg·K}$, $\alpha = 23.7 \times 10^{-6}\ \text{m}^2/\text{s}$;

Table A-6, Saturated water vapor (298K): $v_g = 44.25\ \text{m}^3/\text{kg}$, $h_{fg} = 2443\ \text{kJ/kg}$;(318K): $v_g = 15.52\ \text{m}^3/\text{kg}$;

Table A-8, Air-vapor (1 atm, 298K): $D_{AB} = 0.26 \times 10^{-4}\ \text{m}^2/\text{s}$, D_{AB} (308K) $= 0.26 \times 10^{-4}\ \text{m}^2/\text{s} \times (308/298)^{3/2} = 0.27 \times 10^{-4}\ \text{m}^2/\text{s}$, $Le = \alpha/D_{AB} = 0.88$.

ANALYSIS: Dividing the energy balance on the wick by $\rho_{A,sat}\left(T_\infty\right)$,

$$\frac{T_\infty - T_s}{\rho_{A,sat}\left(T_\infty\right)} = h_{fg}\left[\frac{h_m}{h}\right]\left[\frac{\rho_{A,sat}\left(T_s\right)}{\rho_{A,sat}\left(T_\infty\right)} - \frac{\rho_{A,\infty}}{\rho_{A,sat}\left(T_\infty\right)}\right].$$

With $\left[\rho_{A,\infty}/\rho_{A,sat}\left(T_\infty\right)\right] \approx \phi_\infty$ for a perfect gas and h/h_m obtained from the heat and mass transfer analogy,

$$\phi_\infty = \frac{\rho_{A,sat}\left(T_s\right)}{\rho_{A,sat}\left(T_\infty\right)} - \frac{\rho\, c_p}{Le^{2/3}\rho_{A,sat}\left(T_\infty\right)h_{fg}}\left(T_\infty - T_s\right).$$

where

$$\frac{\rho_{A,sat}\left(T_s\right)}{\rho_{A,sat}\left(T_\infty\right)} = \frac{v_g\left(T_\infty\right)}{v_g\left(T_s\right)} = \frac{15.52}{44.25} = 0.351$$

$$\rho_{A,sat}\left(T_\infty\right) = \left(15.52\ \text{m}^3/\text{kg}\right)^{-1} = 0.064\ \text{kg/m}^3.$$

Hence,

$$\phi_\infty = 0.351 - \frac{1.135\ \text{kg/m}^3\ \left(1007\ \text{J/kg·K}\right)}{\left(0.88\right)^{2/3}\ 0.064\ \text{kg/m}^3\ \left(2.443\times10^6\ \text{J/kg}\right)}\left(45-25\right)\text{K}$$

$$\phi_\infty = 0.351 - 0.159 = 0.192.$$